Global Biogeochemical Cycles

Supporting Information for

Partitioning N_2O Emissions within the US Corn Belt using an Inverse Modeling Approach

Z. Chen[1], T. J. Griffis[1], D. B. Millet[1], J. Wood[1], X. Lee[2], J. M. Baker[1,3], K. Xiao[1], P. A. Turner[1], M. Chen[1], J. Zobitz[4], and K. C. Wells[1]

[1] Department of Soil, Water, and Climate, University of Minnesota, Saint Paul, MN, USA

[2] School of Forestry and Environmental Studies, Yale University, New Haven, CT, USA

[3] United States Department of Agriculture, Agricultural Research Service, Saint Paul, MN, USA

[4] Department of Mathematics, Augsburg College, Minneapolis, MN, USA

Contents of this file

Introduction

This supporting information provides additional details related to the Bayesian inversion.

Text S1. Solutions of Bayesian Inversion

$$y = K\Gamma + \varepsilon \quad \text{(S1)}$$

Where y is the observed minus background mixing ratios, the columns of *K* correspond to the simulated mixing ratios for each of the source types being optimized, and Γ consists of the *a posteriori* scale factors for the 7 source types.

Γ_1 is the scaling factor for *dirA*;

Γ_2 is the scaling factor for *indA*;

Γ_3 is the scaling factor for *waste*;

Γ_4 is the scaling factor for *industry*;

Γ_5 is the scaling factor for *energy*;

Γ_6 is the scaling factor for *natsoil*;

Γ_7 is the scaling factor for *BB*.

Since the Bayesian inversion is conducted each month for each inversion, there are 24×30=720 hourly observed and simulated mixing ratios. The dimension of the matrix K is 720×7 .The cost function $J(\Gamma)$ used to determine Γ is Equation 2. The *maximum a posteriori* (MAP) solution of Γ *is* to minimize the cost function $J(\Gamma)$, S_ε is the observational error covariance matrix, the matrix equals $\sigma_0 \cdot I$, where I is the identity matrix. We are assuming the observational error (σ_0) is constructed and calculated via quadrative, $\sigma_o = \sqrt{\sigma_m{}^2 + \sigma_p{}^2 + \sigma_{mh}{}^2 + \sigma_b{}^2}$, where σ_m is the measurement error, σ_p is the error from particles, σ_{mh} is the error from simulated mixing height, σ_b is the error from background mixing ratios , in a unit of ppb. For simplicity, these individual errors are independent.

S_a is the *a priori* error covariance matrix, it's a diagonal matrix. Along the diagonal, the percentage uncertainty is given corresponding to *the a priori* uncertainty of each source type.